## Homework Solutions

## Chapter 8

## Central Limit Theorem

(a) Draw a tree diagram three levels deep. Label each "yes" with 0.25 and each "no" with 0.75 . Then compute the sampling distribution of $\hat{p}$.


From the diagram, we can calculate the probabilities of the possible values of $\hat{p}$.

| Responses | No. of Yes's | $\hat{p}$ | Probability |
| :---: | :---: | :---: | :--- |
| YYY | 3 | 1 | $(0.25)^{3}=0.0156$ |
| YYN | 2 | $2 / 3$ | $(0.25)^{2}(0.75)=0.0469$ |
| YNY | 2 | $2 / 3$ | $(0.25)^{2}(0.75)=0.0469$ |
| YNN | 1 | $1 / 3$ | $(0.25)(0.75)^{2}=0.1406$ |
| NYY | 2 | $2 / 3$ | $(0.25)^{2}(0.75)=0.0469$ |
| NYN | 1 | $1 / 3$ | $(0.25)(0.75)^{2}=0.1406$ |
| NNY | 1 | $1 / 3$ | $(0.25)(0.75)^{2}=0.1406$ |
| NNN | 0 | 0 | $(0.75)^{3}=0.4219$ |

Finally, summarize this in a table.

| $\hat{p}$ | Probability |
| :---: | :---: |
| 0 | 0.4219 |
| $1 / 3$ | 0.4219 |
| $2 / 3$ | 0.1406 |
| 1 | 0.0156 |

(b) According to the Central Limit Theorem, $\mu_{\hat{p}}=p=0.25$ and

$$
\begin{aligned}
\sigma_{\hat{p}} & =\sqrt{\frac{p(1-p)}{n}} \\
& =\sqrt{\frac{(0.25)(0.75)}{3}} \\
& =0.25 .
\end{aligned}
$$

